Activity description

The Nuffield resource '**Annual percentage rate'** shows students how to find the APR of a loan in the simplest case where the loan is repaid in a single instalment. This resource shows students how to check and then find the APR in more difficult cases when a loan or debt is repaid in several annual instalments.

Suitability

Level 3 (Advanced)

Time

2–4 hours, depending on the ability of the group and the degree of accuracy to which the APR is calculated.

Resources

Student information sheet, worksheet
Optional: slideshow, spreadsheet

Equipment

Calculators

Key mathematical language

APR (Annual Percentage Rate), loan, instalment, interval, bisection, dp, upper bound, lower bound.

Notes on the activity

The slideshow can be used to introduce the activity and the formula used for APR when there is more than one instalment. Answers to the 'Think about' questions are given below:

For 3 annual instalments $C = \frac{A_1}{1+i} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3}$

For 5 annual instalments $C = \frac{A_1}{1+i} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \frac{A_4}{(1+i)^4} + \frac{A_5}{(1+i)^5}$

The symbol ∑ means 'the sum of'

The values £1823.15, £1661.95 and £1514.99 represent the present values of the 3 future instalments.

At the end of the example in section A, students are challenged to explain how they could be certain that 9.7% is correct to 1 decimal place. The solution, using the lower and upper bounds of 0.097 to 1 dp, is given below:

When $i = 0.0965$ $C = \frac{2000}{1.0965} + \frac{2000}{1.0965^2} + \frac{2000}{1.0965^3} = 5005$

When $i = 0.0975$ $C = \frac{2000}{1.0975} + \frac{2000}{1.0975^2} + \frac{2000}{1.0975^3} = 4996$

As the first of these gives a loan value above £5000 and the second gives a loan value below £5000, the value of i must lie between 0.0965 and 0.0975. So the APR must lie between 9.65% and 9.75% , that is the APR is 9.7% correct to 1 decimal place.

During the activity

Students could be encouraged to work in pairs.

Points for discussion

A spreadsheet could be set up to calculate the APR where there is more than one instalment. Ask students to suggest how to do this.

The accompanying spreadsheet gives two worksheets showing how different spreadsheet formulae can be used to find the APR in the example in section B. This can be used to demonstrate to students how a spreadsheet can be used to find an APR or students, can use it themselves to check the answers to the questions in 'Try these B Finding a value for the APR'.

A1	A2	A3	A4	C
£7,500.00	£7,500.00	£8,000.00	£8,000.00	£24,000.00

LB(i)	UB(i)	Midpoint (i)	Value of C	Comparison
0.06	0.12	0.09	£25,038.20	Too high
0.09	0.12	0.105	£24,224.89	Too high
0.105	0.12	0.1125	£23,834.24	Too low
0.105	0.1125	0.10875	£24,028.27	Too high
0.10875	0.1125	0.110625	£23,930.93	Too low
0.10875	0.110625	0.1096875	£23,979.52	Too low
0.10875	0.1096875	0.10921875	£24,003.87	Too high
0.10921875	0.1096875	0.10945313	£23,991.69	Too low
0.10921875	0.10945313	0.10933594	£23,997.78	Too low
0.10921875	0.10933594	0.10927734	£24,000.83	Too high
0.10927734	0.10933594	0.10930664	£23,999.30	Too low

Examples of the spreadsheet formulae needed are given below:

Formula for mid-point of an interval is: =(A5+B5)/2

Formula for the value of C is:
=$A$2/(1+C5)+$B$2/(1+C5)^2+$C$2/(1+C5)^3+$D$2/(1+C5)^4

Extensions

Examples of APRs in more complex cases can be found in *Credit charges and APR* available from the Office of Fair Trading (www.oft.gov.uk).

You could ask students to check values of APR given in real advertisements for credit cards and mortgages as well as loans.

Answers

Try these A

Students can check the suggested APR values by substituting given values of i into an equation of the form:

$$C = \frac{A_1}{1+i} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \frac{A_4}{(1+i)^4}$$

Try these B

Students can find the APR by using the interval bisection method to solve the general equation for i.

1b 10%

2b 6.1%

3b 14.5%

4a 11% **b** 11.2%

5a 13% **b** 12.6%

6 13.6%

7 9.0%

8a Method A £13 500; Method B £14 000

b Method A 16.0%; Method B 15.0%

c i A borrower may prefer Method A because it keeps repayments costs to a minimum.

ii A borrower may prefer Method B because it spreads the repayments over a longer time, and the size of each instalment is lower.